# Measuring Mountains

Name_____

The Englishman Who Went Up a Hill But Came Down a Mountain
(1995 Miramax Studios' movie starring Hugh Grant)
This 1917 story tells of a small village in Wales, that prides itself on being the location of the "first mountain in Wales", traveling south to north. Two English cartographers arrive, measure the mountain, and deem it to be a "hill". Refusing to lose yet one more thing during war time, the inhabitants heroically set out to increase the height of their beloved Ffynnon Garw ("fair in ga roo") to mountain status. While the name of the mountain is fictitious, the story is supposedly a true tale.

1. To qualify as a mountain in the eyes of the British Government, a peak must be 1000 feet in altitude. The cartographers say that they will use two existing hills (Newton Beacon and Whitchurch Hill) to determine the height of Ffynnon Garw. Given the information shown in the diagram, determine the height of Ffynnon Garw which led to its classification as a "hill". This is a two step triangulation process.
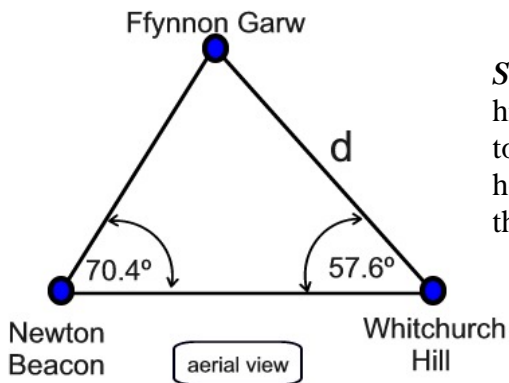
Step 1: From each hill, the angle of Ffynnon Garw to the other hill is measured. The horizontal distance from Newton Beacon to Whitchurch Hill is 1840 feet. Use this information to find the horizontal distance, **d**, from Ffynnon Garw to Whitchurch Hill to the *nearest foot*.
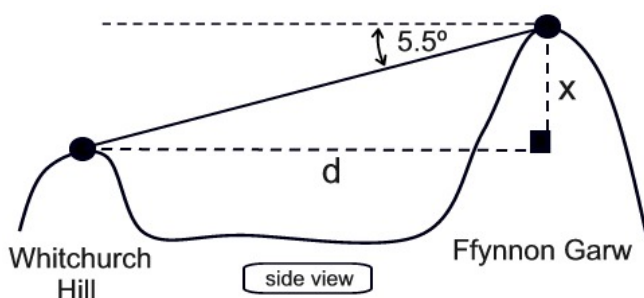

Step 2: The altitude of Whitchurch Hill is known to be 772 feet. From the top of Ffynnon Garw, the angle of depression of the top of Whitchurch Hill is 5.5°.
a) Find the vertical height, **x**, of Ffynnon Garw (above the height of Whitchurch Hill) to the *nearest foot.*
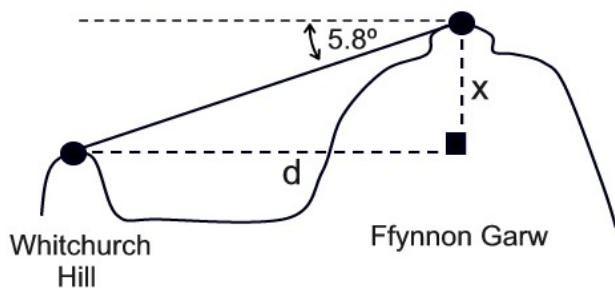b) Is Ffynnon Garw a mountain? _____

2. The older cartographer, George Carrad, is concerned that their motorcar is not operational. (Perhaps it has something to do with the sugar that was added to the gas tank. The villagers were desperate to detain the map makers until they could increase the height of Ffynnon Garw and get a second reading.) As Carrad helps to push the car toward the garage, he determines that the angle of elevation of Morgan the Goat's refreshment stand, from point G, is 10°. After 5 additional minutes of pushing the car, he finds the angle of elevation, from point C, to be 18°. The car is being slowly pushed at 50 feet per minute on a level road. According the Carrad's measurements, what is the height of Morgan the Goat's refreshment stand, to the *nearest foot,* above the level road?

Morgan the Goat's Refreshment Stand
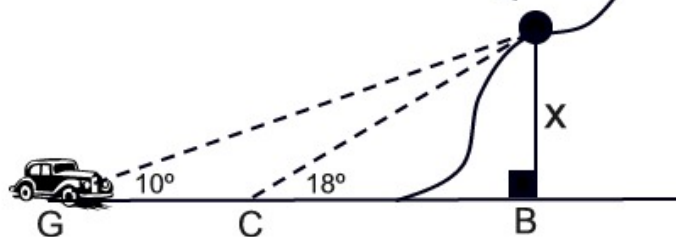
As the villagers work to raise the height of Ffynnon Garw, innkeeper Morgan the Goat sets up a for-profit refreshment stand on the side of the hill.

X

10°   18°

G   C   B

3. After piling dirt on the top of the hill, the villagers eagerly await the next reading on the height of Ffynnon Garw.   A new reading from the summit of Ffynnon Garw shows the angle of depression of the top of Whitchurch Hill to be 5.8°.   Find x, to the nearest foot. Has the beloved "hill" reached "mountain" status? _____

5.8°

X

d

Whitchurch
Hill

Ffynnon Garw

4. As the stress of hauling dirt takes its toll on elderly Reverend Jones, his dying request is to be buried at the peak of Ffynnon Garw so that his burial mound will increase the mountain's height. What is the altitude of Ffynnon Garw, to the *nearest foot,* after the burial of Reverend Jones?

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The angle of depression from the top of the burial mound to the top of Whitchurch Hill is now 6º.
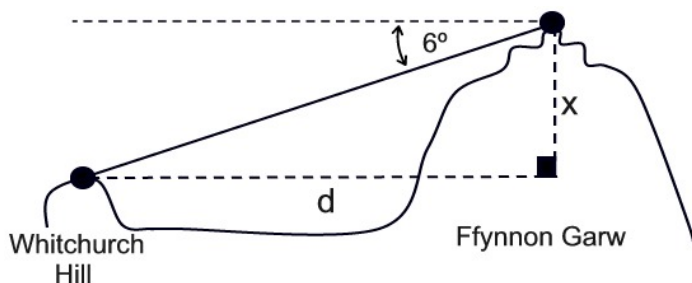
5. Finding heights can also be accomplished using similar triangles. The villagers placed a flag pole at the top of the mountain to gauge the amount of dirt necessary to raise the mountains height. On level ground, point A is 24 feet from the base of the flagpole (CA = 24) and 12 feet from the base of a nearby post (BA = 12). If the post ($\overline{EB}$) is known to be 10 feet tall, how tall is the flagpole ($\overline{DC}$) ?
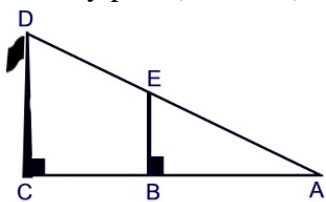
6. As viewed from the pub, Ffynnon Garw appears to be a triangular shape, with a few ragged edges. Drawing imaginary lines from the base points *A* and *B* on level ground to the peak, *P*, a triangle is formed. A power wire runs up the slope from *A* to *P,* a distance of 545 feet, and makes an angle of 25º with the level ground. The angle measured at point *B* is 40º. If the power company has a roll of wire measuring no more than 365 feet, does the power company have enough cable to go down the slope from *P* to *B*?
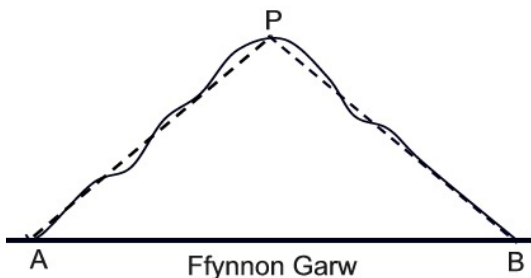
7. The method used in problem 5 could be described as utilizing slopes since it makes use of changes in vertical and horizontal distances of a line. The concept of slope can be applied to finding the height of a mountain by using the slopes of the lines of sight. Standing at point D, the line of sight of the top of the mountain, A, is along $\overrightarrow{DA}$. Similarly, the line of sight from point C is along $\overrightarrow{CA}$.

A formula for finding the height using slopes:

$$h = x \bullet \left(\frac{1}{\dfrac{1}{m_{\overrightarrow{DA}}} - \dfrac{1}{m_{\overrightarrow{CA}}}} \right)$$
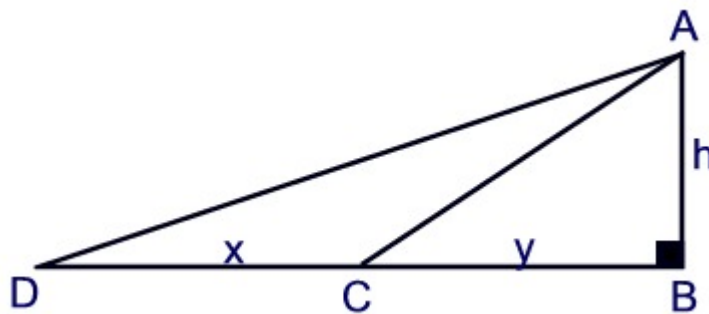
a. Re-write this formula so that it does not contain a complex fraction.

b. Represent the slopes of $\overrightarrow{DA}$ and $\overrightarrow{CA}$ using the lettering in the diagram.

c. Substitute your answers to part b into the formula and show that the formula is true.